# Exact hairy black brane solutions in $AdS_5$ and holographic RG flows

$^{(1)}$Andrés Aceña, $^{(2)}$Andrés Anabalón, and $^{(3)}$Dumitru Astefanesei

$^{(1)}$*Instituto de Ciencias Básicas, Universidad Nacional de Cuyo, Mendoza, Argentina.*
$^{(2)}$*Departamento de Ciencias, Facultad de Artes Liberales y*
*Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Viña del Mar.*
$^{(3)}$ *Instituto de Física, Pontificia Universidad Católica de Valparaíso,*
*Casilla 4059, Valparaíso, Chile*
acena.andres@gmail.com, andres.anabalon@uai.cl, dumitru.astefanesei@ucv.cl

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## Abstract

We construct a general class of *exact, regular* black hole solutions with toroidal horizon topology in 5-dimensional AdS gravity with a self-interacting scalar field. Due to the non-trivial backreaction of the scalar field, the no-hair theorems can be evaded so that an event horizon can be formed. The scalar field is regular everywhere outside the curvature singularity and it vanishes at the boundary where the potential is finite. We study the properties of these black holes in the context of AdS/CFT duality and comment on the dual operators, which saturate the unitarity bound. We present exact expressions for the beta-function and construct a c-function that characterizes the RG flow.

## 1 Introduction.

Motivated by AdS/CFT duality [1], there has been extensive work constructing solutions in Anti de Sitter (AdS) spacetime. The finite temperature dynamics of holographic field theories can be related to the thermodynamics of black holes on the gravity side. Of particular interest are black hole solutions with scalar hair (see, e.g., [2, 3, 4, 5, 6, 7, 8, 9, 10] for some recent applications). However, many of these solutions (for non-trivial scalar potentials) were generated numerically only[1] and so the physics of these solutions in the context of holography can be just partially investigated.

In this letter, we present a class of *exact*, toroidal black hole solutions in AdS$_5$ gravity with a self-interacting scalar field — the non-trivial backreaction of the scalar field can produce an event horizon. To the best of our knowledge, this is the first example of a 5-dimensional exact, regular, neutral, hairy black hole solution. These solutions are important for understanding the no-hair theorems when the horizon does not have spherical topology (see, e.g., [11]) and are constructed by using the method of [12] (see, also, [13]).

Within AdS/CFT duality, the physics of this family of solutions is very rich. The classical (super)gravity regime corresponds to the large-N, strong 't Hooft coupling limit of the gauge theory. Using exact AdS gravity backgrounds[2], it is possible to investigate in detail theories that have some similar features with four-dimensional QCD, e.g. confinement/deconfinement transitions [14], non-trivial speed of sounds, bulk viscosities [15], RG flows ($\beta$-functions) [16, 17, 18], hydrodynamic properties

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[1]In fact, in many relevant examples for holography, by imposing constraints on the form of the scalar potential just the asymptotic and near horizon expansions of the metric are presented.

[2]By including additional background fields, which maintain the asymptotic boundary behaviour, the conformal symmetry can be broken.

[19], et cetera. The existence of exact hairy black branes in AdS opens the exciting possibility of investigating the strong regime of certain field theories in a well controlled setting.

Interestingly enough, a part of the scalar potential we consider is controlled by a tunable parameter, $\alpha$, so that we can obtain exact domain wall solutions when $\alpha = 0$ and exact planar black holes when $\alpha \neq 0$. This allows to explore how the extended literature on domain walls and fake supergravity potentials apply to this case. Since the black hole solutions are analytic, we obtain exact beta-functions. We would like to point out that the expressions for the domain wall and black hole beta-functions coincide, though the physics is obviously different.

For tachyonic scalars, it was shown in [20] that the slow $a$-branch[3] contributes to the boundary charges. The range of the mass was chosen so that the slow branch satisfies both, the Breitenlohner-Freedman and unitarity bounds: $-4 \leq m^2 l^2 < -3$, where $l$ is the $AdS_5$ radius. In [20], it was argued that, when the unitarity bound is violated $\Delta_- \leq 1$, the Klein-Gordon inner product is divergent when evaluated for these configurations. However, the Klein Gordon inner product is well defined only for solutions of the *linearized* Klein-Gordon field equation, namely perturbations over the configurations that we are considering here. Indeed, these linearized perturbation seems to be normalizable only if Dirichlet boundary conditions are imposed on them [21].

In the context of AdS/CFT, a simple realization of renormalization group flows is given by turning on a scalar field dual to a relevant operator deformation and solving the coupled equations of motion in the bulk. One important reason for choosing our scalar potential was to construct exact solutions compatible with scalars with mass $m^2 = -3/l^2$. The dual theories correspond to deformations by operators that saturate the unitarity bound. Unitarity bounds help to establish that a certain field decouples from the rest of the theory in the infrared and this is particularly interesting in the light of recent work [21].

Last, but not least, the black hole solutions we found are also important because they provide concrete examples of solutions with vanishing horizon area that can be 'dressed' with a horizon due to the new non-trivial terms/corrections in the potential.

We will present the thermodynamical properties of our solutions and also discuss in detail some of their interesting 'holographic' features.

## 2 The setup

The action we are interested in is

$$I[g_{\mu\nu}, \phi] = \frac{1}{2\kappa} \int_M d^5x \sqrt{-g} \left[ R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] - \frac{1}{\kappa} \int_{\partial M} d^4x \sqrt{-\gamma} K \tag{1}$$

where $V(\phi)$ is the scalar potential and we use the convention $\kappa = 8\pi G_N$. Here, $K$ is the trace of the extrinsic curvature of the boundary $\partial M$ as embedded in $M$ and $\gamma_{ab}$ is the induced metric on the boundary. Since we set $c = 1 = \hbar$, $[\kappa] = M_P^{-3}$ where $M_P$ is the five-dimensional Planck scale.

The equations of motion for the metric and dilaton are

$$E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} T_{\mu\nu}^\phi = 0 , \quad \frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right) - \frac{\partial V}{\partial \phi} = 0 \tag{2}$$

where the stress tensors of the matter fields is

$$T_{\mu\nu}^\phi = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left[ \frac{1}{2} (\partial \phi)^2 + V(\phi) \right] \tag{3}$$

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[3] The asymptotic form of the scalar field is $\phi = \frac{a}{r^{\Delta_-}} + \frac{b}{r^{\Delta_+}}$, where $\Delta_\pm = 2 \left( 1 \pm \sqrt{1 + \frac{l^2 m^2}{4}} \right)$ and $l$ is the AdS radius. The slow branch is the one controlled by $\Delta_-$.

For simplifying our analysis, we use the following ansatz for the metric:

$$ds^2 = \Omega(x)\left[-f(x)dt^2 + \frac{\eta^2 dx^2}{f(x)} + d\Sigma^2\right] \tag{4}$$

where the parameter $\eta$ was introduced to obtain a dimensionless radial coordinate $x$, $\Omega(x)$ is the conformal factor, and $d\Sigma^2 = \sum_{a=1}^{3} dx_a^2$.

There are three independent (combinations of) equations of motion. Two of them are

$$E_t^t - E_r^r = 0 \implies \phi'^2 = \frac{9\left(\Omega'\right)^2 - 6\Omega\Omega''}{2\Omega^2} \tag{5}$$

$$E_t^t - E_a^a = 0 \implies 2\Omega F'' + 3\Omega' F' = 0 \tag{6}$$

where the derivatives are with respect to $x$. The remaining equation is more complicated, but it is worth emphasizing that the potential can be obtained by solving it.

## 3   Solutions

As in [12], we choose the conformal factor

$$\Omega(x) = \frac{\nu^2 x^{\nu-1}}{\eta^2\left(x^\nu - 1\right)^2} \tag{7}$$

so that we can solve the dilaton equation of motion to get

$$\phi = l_\nu^{-1}\ln(x) \quad , \quad l_\nu^{-1} = \sqrt{\frac{3(\nu^2 - 1)}{2}} \tag{8}$$

A canonically normalized scalar field has the dimension $[l_\nu] = [L^{\frac{3}{2}}]$, but with our normalization the scalar field is dimensionless. The parameter $\nu$ labels different hairy solutions.

The other metric function is

$$f(x) = -\frac{\Lambda}{6} + \alpha\left[\frac{4}{3\left(\nu^2 - 25\right)\left(9\nu^2 - 25\right)} + \frac{x^{\frac{5}{2}}}{12\nu^3}\left(\frac{x^{\frac{3\nu}{2}}}{3(3\nu + 5)} + \frac{x^{-\frac{3\nu}{2}}}{3(3\nu - 5)} - \frac{x^{\frac{\nu}{2}}}{(\nu + 5)} - \frac{x^{-\frac{\nu}{2}}}{(\nu - 5)}\right)\right] \tag{9}$$

and the dilaton potential is

$$V(\phi) = \frac{(9\nu^2 - 25)e^{-\phi l_\nu}}{4\nu^2}\left[\Lambda - \frac{8\alpha}{(\nu^2 - 25)(9\nu^2 - 25)}\right]\left[\frac{(\nu + 1)}{2(3\nu - 5)}e^{\nu\phi l_\nu} + \frac{(\nu - 1)}{2(3\nu + 5)}e^{-\nu\phi l_\nu} + \frac{5(\nu^2 - 1)}{(9\nu^2 - 25)}\right]$$

$$+ \frac{\alpha e^{\frac{3}{2}\phi l_\nu}}{2\nu^3}\left[\frac{5(\nu^2 - 1)}{\nu^2 - 25}\left(\frac{e^{\frac{\nu}{2}\phi l_\nu}}{3\nu + 5} + \frac{e^{-\frac{\nu}{2}\phi l_\nu}}{3\nu - 5}\right) + \frac{1}{3}\left(\frac{(\nu - 1)e^{\frac{3\nu}{2}\phi l_\nu}}{(\nu + 5)(3\nu + 5)} + \frac{(\nu + 1)e^{-\frac{3\nu}{2}\phi l_\nu}}{(\nu - 5)(3\nu - 5)}\right)\right] \tag{10}$$

At this point we observe that the potential has two parts, one which is controlled by the cosmological constant $\Lambda = -\frac{6}{l^2}$ and the other by $\alpha$ that is an arbitrary parameter with the same dimension as $\Lambda$, namely $[\alpha] = [L^{-2}]$.

At the boundary, $x = 1$, the scalar field vanishes, the potential is $V = 2\Lambda$ and, moreover, $\frac{\partial V}{\partial \phi}\Big|_{\phi=0} = 0$, $\frac{\partial^2 V}{\partial \phi^2}\Big|_{\phi=0} = -\frac{3}{l^2}$. Therefore, the theory has a standard AdS vacuum at $\phi = 0$. The rank of the

coordinate $x$ can be taken to be either $x \in (0, 1]$ or $x \in [1, \infty)$. The scalar field is negative in the first case, but positive in the other range. Since the dilaton potential has no obvious symmetry, this allows to cover it completely. The singularities of the metric (and the scalar field) are at $x = 0$ and $x = \infty$, but they are enclosed by an event horizon.

It is easy to see that, for any value of $\Lambda$, $\nu$, and $x_+$, there is an $\alpha$ such that $f(x_+) = 0$. Indeed, this simply follows from the fact that $f(x)$ is linear in $\alpha$. Therefore, there are black holes in an open set of the parameter space. Moreover, a simple inspection of the function (9) shows that is regular for every positive $x \neq 0$ and $x \neq \infty$.

We conclude this section by observing that, due to the non-trivial potential and the backreaction of the scalar field, there exist *regular* hairy black hole solutions and the metric depends on the parameter $\alpha$ as expected.

## 4    Thermodynamics

In this section we describe in detail the thermodynamical properties of our solutions and show that the first law is satisfied. We also compute the free energy and comment on the existence of phase transitions.

To gain some intuition, let us first consider the limit for which we can recover the usual no-hairy planar black hole. This is possible in the limit $\nu = 1$ when the conformal factor becomes $\Omega(x) = [\eta(x - 1)]^{-2}$. To obtain the usual planar coordinates, we should match the factors of $d\Sigma^2$ so that $\Omega(x) = r^2$ and so we get (for the positive branch)

$$x = 1 + \frac{1}{\eta r} \tag{11}$$

and for $\nu = 1$ we obtain

$$f\Omega = r^2 \left[ -\frac{\Lambda}{6} + \left( \frac{1}{3} \right) \left( \frac{\alpha}{96\eta^4} \right) \frac{1}{r^4} \right] = \frac{r^2}{l^2} + \frac{1}{3} \left( \frac{\alpha}{96\eta^4} \right) \frac{1}{r^4} \tag{12}$$

The mass of the planar black hole was computed in [22] by using counterterms :

$$M = \frac{3}{16\pi G_N} m V_3 = -\left( \frac{1}{16\pi G_N} \right) \frac{\alpha}{96\eta^4} V_3 \tag{13}$$

where $m$ is the mass parameter of the planar black hole and $V_3$ is the infinite volume of the space with the metric $d\Sigma^2$.

Since in the planar case there is no Casimir energy, it is more convenient when the scalars are turned on to use the method of Ashtekar, Das, and Magnon [23]. Interestingly enough, the mass has the same form as for the planar black hole discussed above:

$$M_{hairy} = -\left( \frac{1}{16\pi G_N} \right) \frac{\alpha}{96\eta^4} V_3 \tag{14}$$

but now the parameter $\alpha$ plays the role of a multiplicative coupling constant in the Lagrangian and can not be eliminated as in the previous case, when just the mass parameter characterizes the solution.

To check the first law, we also need the temperature and entropy of the hairy black hole that are

$$T = \left. \frac{f'(x)}{4\pi\eta} \right|_{x=x_+} = -\frac{\alpha \left| x_+^\nu - 1 \right|^3}{288\pi\eta\nu^3 x_+^{\frac{3}{2}(\nu-1)}} \tag{15}$$

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$$S = \frac{A}{4G_N} = \frac{\Omega(x_+)^{\frac{3}{2}}}{4G_N}V_3 = \frac{\nu^3 x_+^{\frac{3}{2}(\nu-1)}}{4G_N\,\eta^3\left|x_+^\nu - 1\right|^3}V_3 \tag{16}$$

where $x_+$ is the location of the horizon, which is the largest root of $f(x_+) = 0$.

Now, it is easy to check that the first law is satisfied if we work with densities of the relevant physical quantities. It is well known that, with the Wick rotation $t \to i\tau$, the Euclidean path integral yields a thermal partition function. Then, the Euclidean black hole has the interpretation of a saddle-point in this path integral and so the gravity action evaluated for the classical solution is the leading contribution to the free energy:

$$F = I_E T = M - TS = \frac{\alpha}{3^2 2^9 \pi G_N \eta^4}V_3 < 0. \tag{17}$$

As in the no-hairy planar black hole case [24], we found [25] that there is a similar phase transition between the hairy black holes and a hairy AdS soliton that is obtained by a double analytic continuation (as in [26]) from the planar hairy black hole.

## 5   Holography

The AdS/CFT duality is an example of strong/weak coupling duality and so it can be used as a tool for understanding some properties of strongly coupled quantum field theories in terms of AdS black holes physics. It is well known that the 't Hooft coupling ($g_{YM}^2 N \sim g_s N$) controls the perturbative expansion of $U(N)$ gauge theories, where the string coupling[4] is $g_s = e^\phi$. In the gravity side there is a reliable background when the radius of curvature of AdS is large compared to the string scale $l^4/l_s^4 \sim g_s N \gg 1$. Since we should also neglect contributions from string loops, it is necessary that $g_s \ll 1$, which then implies that $N \to \infty$. It is clear now that the regime in which the AdS description of the physics is valid is completely opposite to the regime of perturbative field theory.

In the AdS/CFT context, the CFT on the boundary is the UV fixed point of a 4-dimensional QFT. Deformation of the gauge theory by the addition of relevant operators is one way to reduce its symmetries. Using the gravity side of the correspondence (deformations of $AdS_5$), one can obtain holographic RG flows [16, 18] corresponding to non-conformal field theories. Here, we consider the decoupling limit of AdS/CFT duality at finite temperature [1] when the bulk theory is a black hole:

$$ds^2 = b^2(u)\left[-f(u)dt^2 + d\Sigma^2 + \frac{du^2}{f(u)}\right] \tag{18}$$

and the boundary where the field theory is at the UV fixed point is $u \to 0$.

The running of the gauge coupling is simply a consequence of the dilaton being non-constant. The beta-function of the theory, as a function in terms of the background solution, is

$$\beta(e^\phi) = b(u)\frac{de^{\phi(u)}}{db(u)} \tag{19}$$

Given this definition, it is straightforward to write the beta-function in the coordinates of (4)

$$\beta(e^\phi) = 2\Omega\frac{de^{\phi(x)}}{d\Omega} = 2\frac{\Omega}{\Omega'}\frac{de^{\phi(x)}}{dx} = 2\frac{\Omega}{\Omega'}\frac{dx^{l_\nu^{-1}}}{dx} = -\frac{2}{l_\nu}\frac{e^{\nu\phi l_\nu} - 1}{e^{\nu\phi l_\nu}\nu + e^{\nu\phi l_\nu} + \nu - 1}e^\phi \tag{20}$$

As expected, we can easily see that the $\beta$-function vanishes at the conformal UV fixed point.

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[4]In string theory the couplings are identified with the moduli space of the theory under consideration.

In general, the beta-function is derived from the moduli potential via a superpotential. In our case, since we have exact solutions we can use directly (20). However, for completeness, let us also discuss the case $\alpha = 0$ for which the superpotential can be explicitly obtained. Unlike the $\alpha \neq 0$ case, for $\alpha = 0$ we obtain a domain wall with a naked singularity. The superpotential is

$$W(\phi) = \frac{\sqrt{6}}{2\nu l}\left[(\nu - 1)\, e^{-\frac{(\nu+1)\phi l_\nu}{2}} + (\nu + 1)\, e^{\frac{(\nu-1)\phi l_\nu}{2}}\right] \tag{21}$$

where

$$V(\phi) = 3\left[\frac{dW(\phi)}{d\phi}\right]^2 - 2W(\phi)^2 \tag{22}$$

We can then compute the $\beta$-function from the superpotential as in [17] $\beta(\phi) = -\frac{1}{W(\phi)}\frac{dW(\phi)}{d\phi}$ and see that, indeed, has the same expression as (20) (the $\beta$-function does not depend explicitly on $\alpha$).[5]

Let us now compute the c-function that is an off-shell generalization of the central charge. The central charge counts the number of massless degrees of freedom in the CFT, in other words it counts the ways in which the energy can be transmitted. The coarse graining of a quantum field theory removes the information about the small scales and so, for a QFT RG flow, there should exist a c-function that is decreasing monotonically from the UV regime (large radii in the dual AdS space) to the IR regime (small radii in the gravity dual) of the QFT.

Here, we follow closely [18]. By using the null energy condition (that holds in our case) and the equations of motion, one can show that for an ansatz:

$$ds^2 = -a(r)^2 dt^2 + \frac{dr^2}{c^2(r)} + b^2(r)d\Sigma^2 \tag{23}$$

the c-function is $C(r) = C_0 \frac{a^3}{b'^3 c^3}$, which in our coordinates becomes

$$C(x) = 8C_0 \frac{\Omega^{\frac{9}{2}}}{\Omega'^3} = 8C_0 \frac{\nu^3}{\eta^3} \frac{x^{\frac{3}{2}(\nu+1)}}{(\nu x^\nu + \nu + x^\nu - 1)^3} \tag{24}$$

and has the right properties. The constant is fixed by the entropy of black hole (in the IR regime). For the planar neutral black hole, the conformal radius is $b(r) = r$ and the c-function is constant, the flow is trivial (this corresponds to 'hairless' limit $\nu = 1$).

We see that the c-function is completely determined by the conformal factor and the other function in the metric, $f(x)$, does not play any role. We can understand why since, in the dual description, the black hole is interpreted as a thermal state. Therefore, the c-function should remain unchanged when we excite a finite temperature vacuum in the same theory (with the same degrees of freedom).

We would like to explain why the parameter $\alpha$ controls the physics in deep infrared only. For this, let us expand the potential around the boundary ($\phi = 0$):

$$V(\phi) = -\frac{12}{l^2} + \frac{(\nu^2 - 1)}{l^2}\left(-\frac{9l_\nu^2}{4}\phi^2 + \frac{l_\nu^3}{4}\phi^3 - \frac{3(\nu^2 - 2)l_\nu^4}{16}\phi^4 + \frac{(7\nu^2 - 18)l_\nu^5}{80}\phi^5\right.$$
$$\left. + \frac{(420 - 12\nu^2(3\nu^2 + 5) + 5\alpha l^2)l_\nu^6}{5760}\phi^6 + ...\right) \tag{25}$$

where the first non-trivial contribution of the $\alpha$ term was included.

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[5]In fact, since in [17] the beta function is defined as $\beta = a\frac{d\phi}{da}$ and not with respect to $e^\phi$, the results match up to an $e^\phi$ factor.

Interestingly, it was shown in [20] that the $O(\phi^3) + ...$ terms in the scalar potential are irrelevant to calculating the Hamiltonian generators. Since for our solutions $\alpha$ appears at the 6th order in the expansion, this part of the scalar field potential does not backreact strongly in the boundary, but in the bulk the self-interaction term controlled by $\alpha$ is responsible for the appearance of the horizon.

We change the coordinates

$$\ln x = \frac{1}{\eta r} - \frac{1}{2\eta^2 r^2} - \frac{\nu^2 - 9}{24\eta^3 r^3} + \frac{\nu^2 - 4}{12\eta^4 r^4} + \frac{(9\nu^2 - 25)(\nu^2 - 25)}{1920\eta^5 r^5} \tag{26}$$

so that our metric matches the asymptotic form of [20].

Solving the linearized scalar field equations with constant mass term, there are two independent solutions. Since for our solutions $m^2 l^2 = -3$, we obtain $\Delta_+ = 3$, $\Delta_- = 1$ and so the behaviour of this scalar at the boundary is

$$\phi = \frac{a}{r}(1 + ...) + \frac{b}{r^3}(1 + ...) \tag{27}$$

In the special case $\nu = 3$, the mode corresponding to $\Delta_+ = 3$ vanishes. Depending on the form of the potential, there is a logarithmic term that can appear when $\Delta_+/\Delta_-$ is an integer [20] (see, also, [27]), however it is not present in this solution.

(Super)gravity fields do not scale under 4-dimensional conformal transformations, so $a$ must have dimension 1 and $b$ must have dimension 3 that corresponds to $a$ being a source (i.e. a mass) and $b$ being its VEV (or condensate). Since $m^2 < 0$, the dual operators are relevant and so the backreaction will be so that the metric remains asymptotically AdS, reflecting the conformal fixed point in the UV. One can check this observation in the coordinates of [20] for which we obtain the deviations from the AdS metric at infinity as follows: the change of coordinates (26) brings the metric to standard AdS coordinates. Indeed, the metric takes the form

$$ds^2 = -\frac{r^2}{l^2}dt^2 + \frac{l^2}{r^2}dr^2 + r^2\delta_{mn}dx^m dx^n + h_{\mu\nu}dx^\mu dx^\nu \tag{28}$$

where $h_{\mu\nu}$ are the deviations from AdS. One can then obtain that the fall off of the metric is the one predicted in [20] for the generic case:

$$h_{rr} = -\frac{(\nu^2 - 1)l^2}{4\eta^4 r^4} + O(r^{-5}), \tag{29}$$

$$h_{tt} = -\frac{\alpha}{288\eta^4 r^2} + O(r^{-3}), \tag{30}$$

$$h_{mn} = \frac{(\nu^2 - 9)(4\nu^2 - 9)}{120\eta^5 r^3}\delta_{mn} + O(r^{-4}). \tag{31}$$

We would like to point out that the Ashtekar-Das-Magnon mass is exactly the $r^{-2}$ factor in the lapse function.

# 6 Discussion and future directions

Finding exact solutions of Einstein field equations with (or without) matter sources is a subject of long standing interest. Indeed, exact solutions with non-trivial moduli potentials can be important, in particular, for phenomenological bottom-up approaches in string theory (see, e.g., [15, 28]). Some predictions of the gauge/gravity correspondence may be universal enough as to apply to QCD, at least in certain regimes.

The thermodynamical properties and some holographic features of our solutions were discussed in detail in the previous sections. One important result we have obtained is the existence of a non-trivial RG flow and, for future work, it will be interesting to understand how much the dual field theory 'mimics' QCD as in [15, 28]. In particular, one could compute the speed of sound and bulk viscosity and see which values of the 'hairy parameter' ($\nu$) are interesting. There is also a conjecture of Buchel on a dynamical bound of the bulk viscosity [29] that can be checked with our solutions.

Let us comment now on the limit $\alpha = 0$ that is equivalent with considering a vanishing mass for our solution. We see that the potential is still non-trivial, but in this case we obtain a naked singularity. Once the self-interaction of the scalar field, which is proportional to the parameter $\alpha$ is considered the singularity gets 'dressed' by a horizon and a regular black hole solution is obtained. This is one concrete example for which new terms/corrections of the potential convert singular solutions into regular black holes with finite horizon area.

One possible extension of our work is to include gauge fields. The physics and phase structure are richer in this case and it is possible to investigate the physics at zero temperature due to the existence of extremal black hole solutions. When the scalar field is non-minimally coupled to the gauge fields and the scalar charge is determined by the charges and mass, the hair is referred to as 'secondary'. The scalar charge is not protected by a gauge symmetry and so it is not a conserved charge. It was shown in [30] that the scalar charges, even if not conserved, can appear in the first law of thermodynamics (the interpretation of this result, though, should be taken with caution [31]).

In the presence of gauge fields, we were also able to construct exact solutions in AdS [32].[6] In the extremal limit, since the flow of the moduli is interpreted as an RG flow, the attractor mechanism acts as a no-hair theorem [34].

There are other interesting directions, which can be investigated. For example, using the results in Section 4, it is straightforward to obtain the phase structure of our solutions. When the foliation of Euclidean AdS is $R_3 \times S^1$ ($S^1$ is the Euclidean time circle), the only scale in the system (the temperature) can be scaled out via conformal invariance. Thus, the $\mathcal{N} = 4$ theory on $R_3$ cannot have a phase transition at any nonzero temperature [35]. However, there are first order transitions between these black holes and the corresponding soliton (that is the solution with the minimum energy within the solutions with the same boundary conditions), which is constructed by a double analytic continuation similar with the one in [26]. It is important to emphasize that there also are second order phase transitions similar to the ones discussed in [36]. These results will be presented in a companion paper [25].

To end our discussion we would like to point out that with our method we can also generate exact solutions with scalar and gauge fields when the cosmological constant is positive. These solutions can be described along the lines of [37] (e.g., computing the corresponding c-functions).

# 7   Acknowledgments

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[6]The existence of solutions for which both modes of the scalar are normalizable was numerically proven in [33].

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